6- Quantum Matter

Quantum Information + Condensed Matter Physics

Understanding Quantum Matter and
Quantum Phase Transitions in a deeper way

There are new phases of matter which cannot be distinguished from each other by any local measurements.

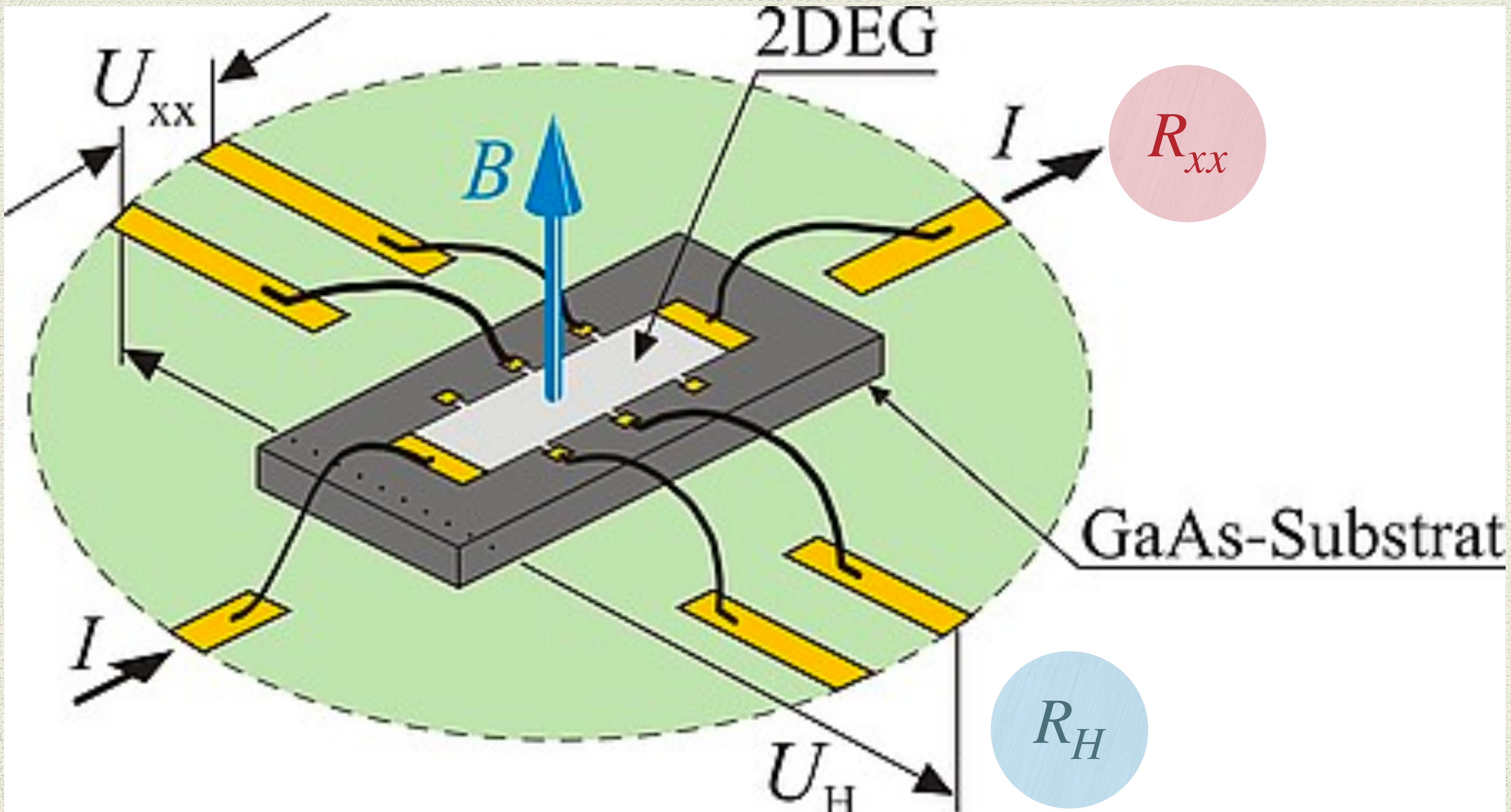
There are new phases are called topological phases.

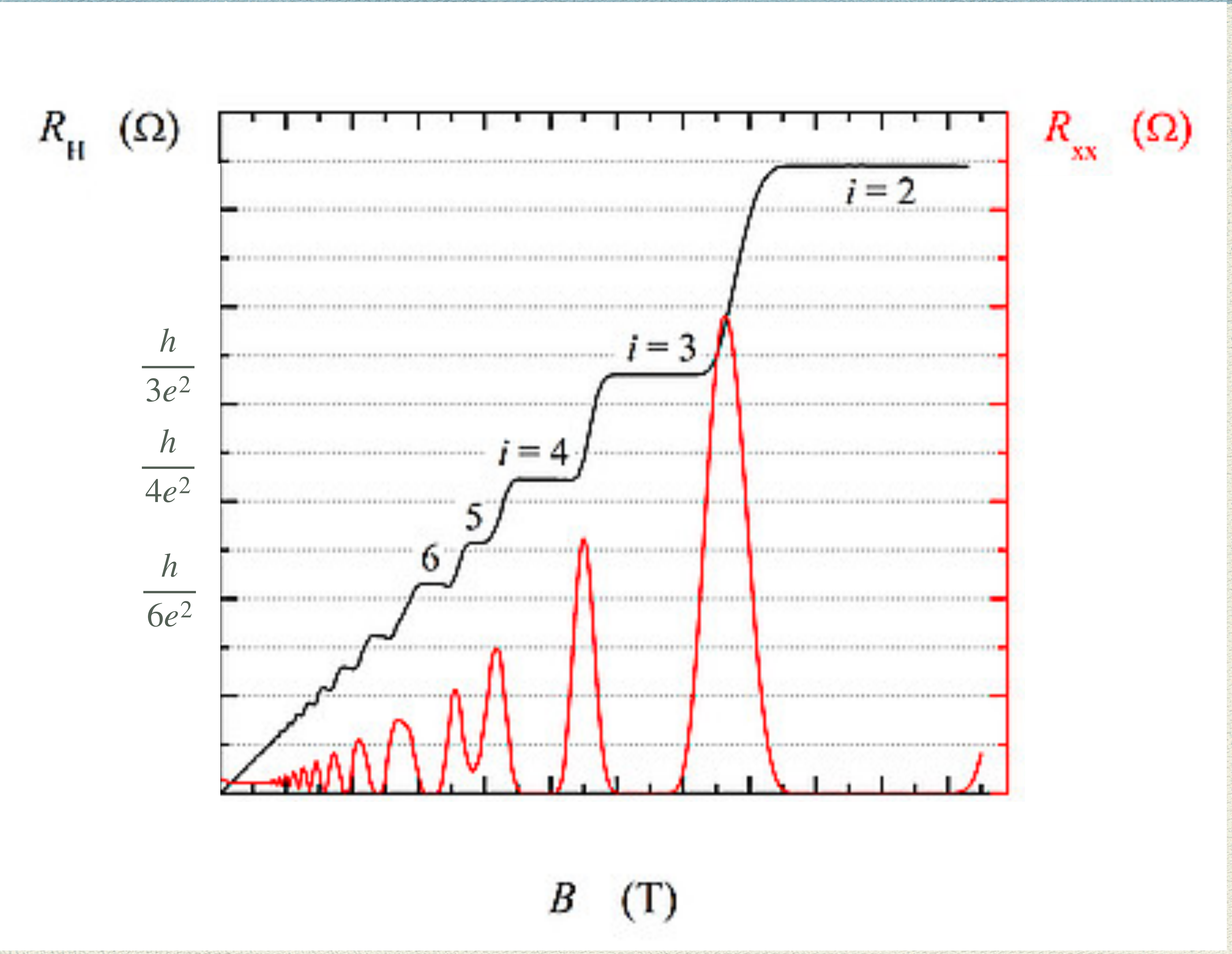
Experimentally the first signs of topological phases
and topological phase transition
appeared in quantum Hall effect.

Topological phases are characterized
by the pattern of long-range entanglement
in many body systems.

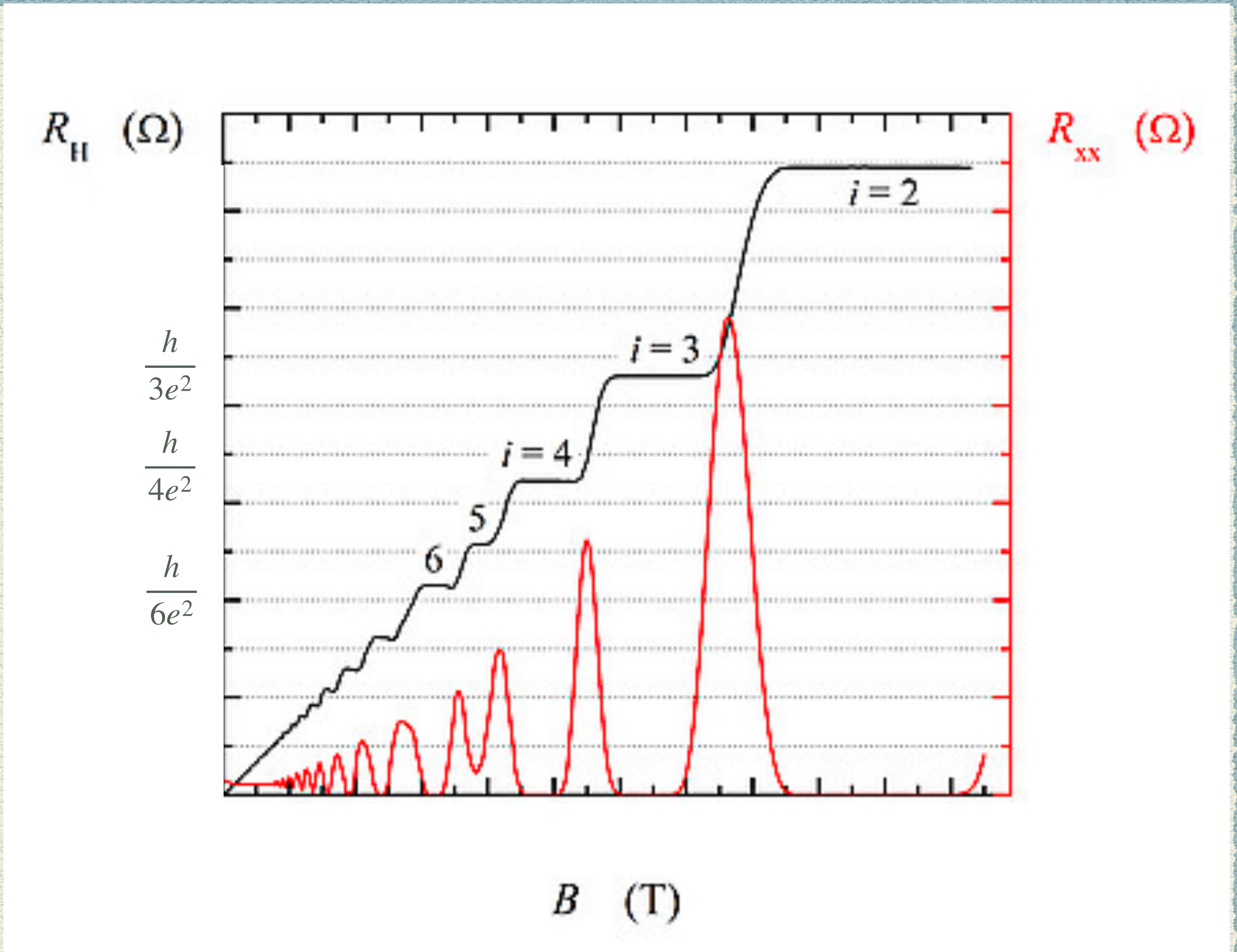
Theoretically hundreds of topological phases are possible.

Quantum Hall effect

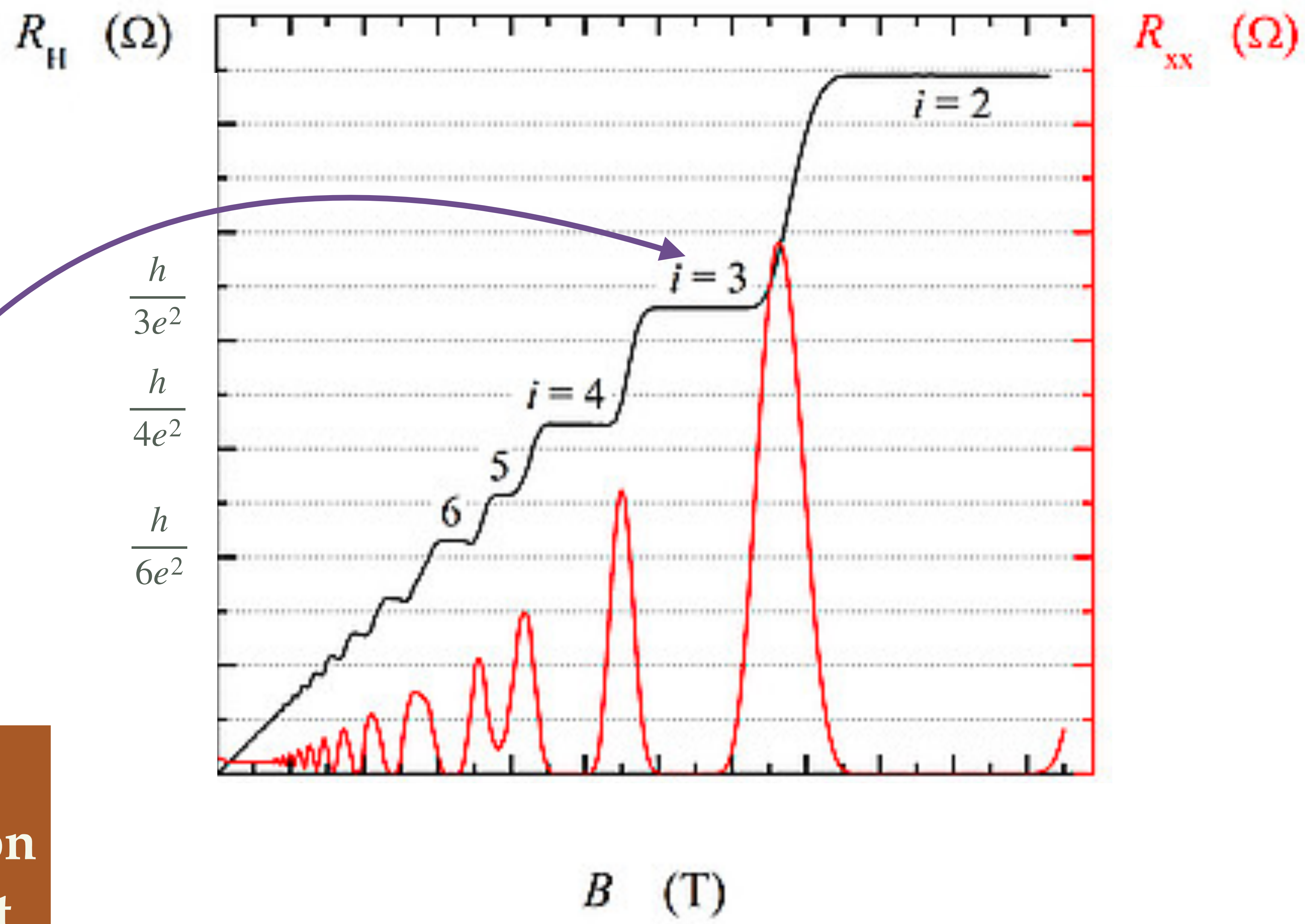




Condensed matter and quantum information (82)



For the first time we see discreteness
or quantum-mess in macroscopic scales.



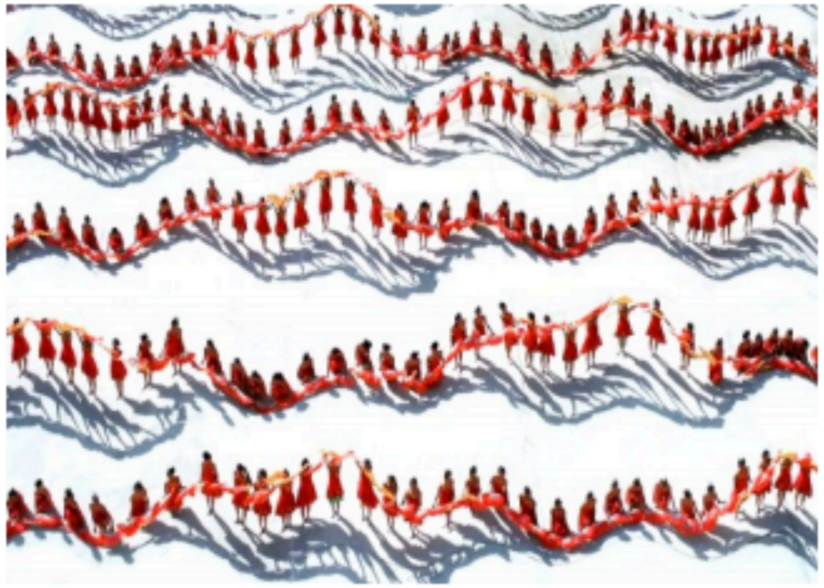
In each of these plateaus, the electron gas is in a different topological phase. Each phase has a different type of long range entanglement.

Long range quantum entanglement

Xiao-Gang Wen

Topological Order: From Long-Range Entangled Quantum Matter
to a Unified Origin of Light and Electrons

FQH state

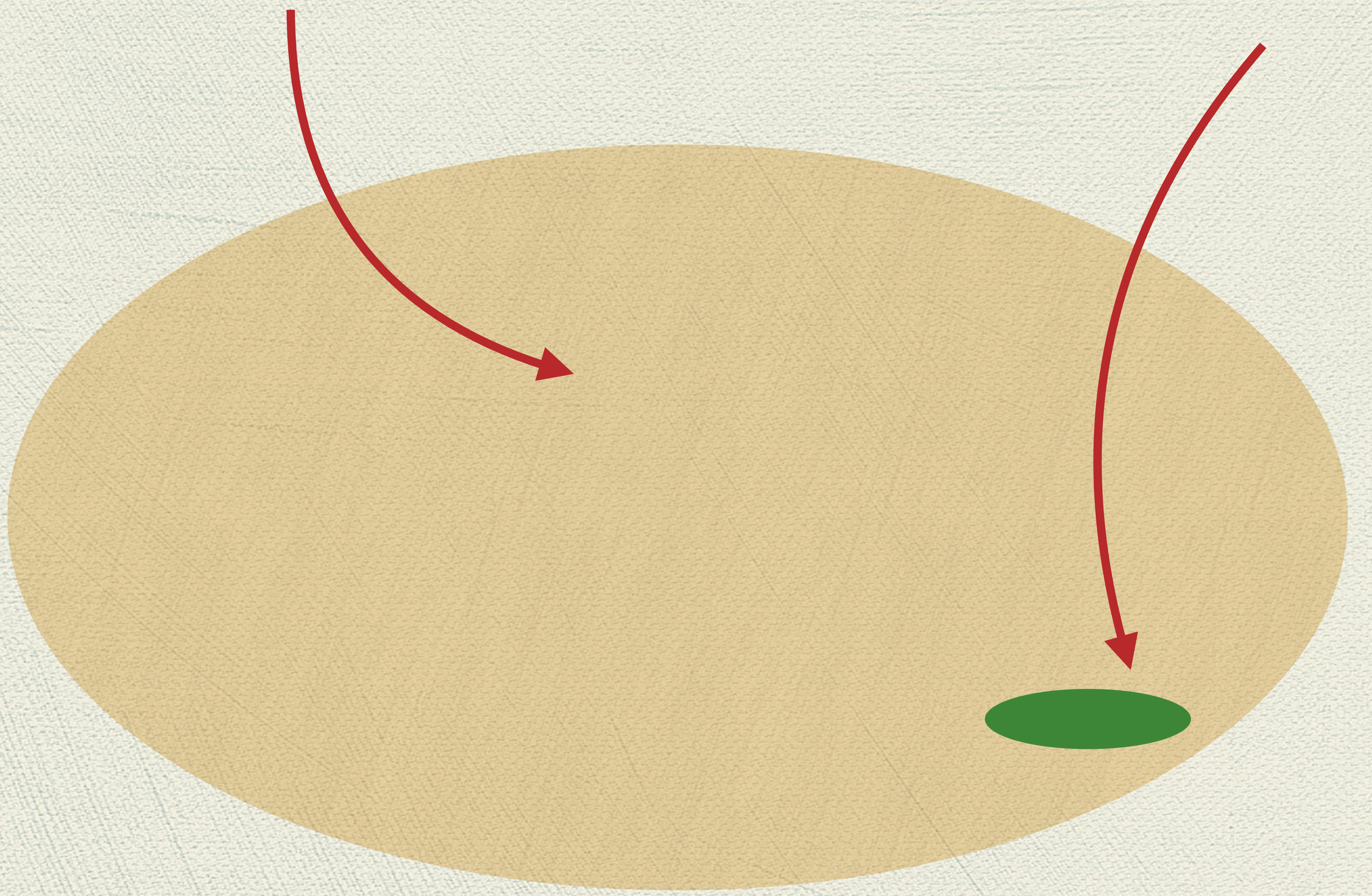


String liquid (spin liquid)

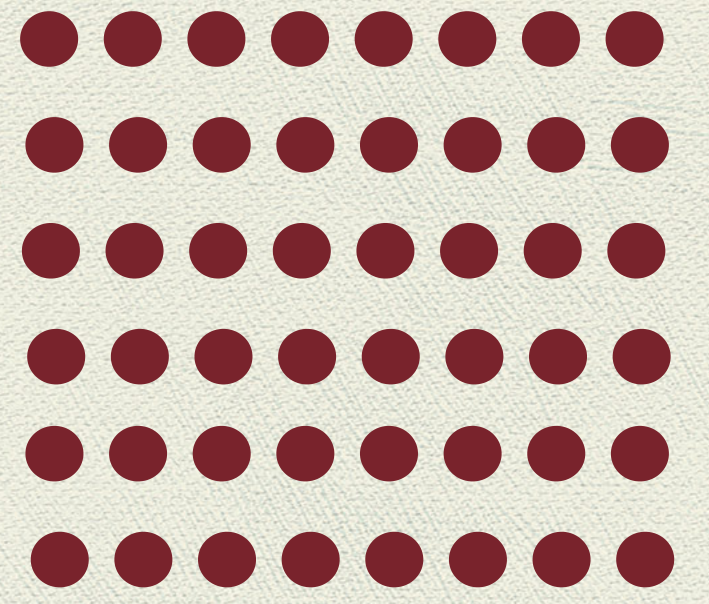
$$|\Psi\rangle = \sum_{i_1, i_2, \cdots i_n} \psi_{i_1, i_2, \cdots i_n} | i_1, i_2, \cdots i_n \rangle$$

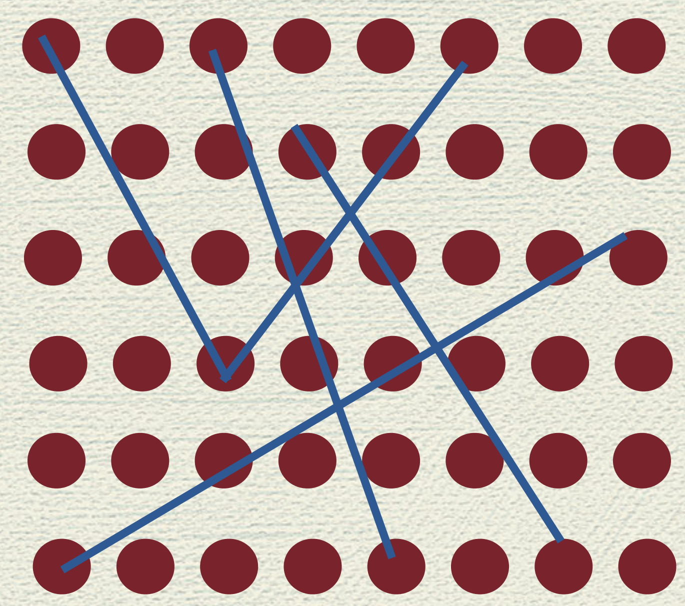
A general state needs 2^n parameters

Some states need $O(n^k)$ parameters.



A very large Hilbert space





$$|\Psi\rangle = |\phi_1\rangle|\phi_2\rangle\cdots\phi_n\rangle$$

$$|\Psi\rangle = \sum_{i_1,i_2,\cdots i_n} \psi_{i_1,i_2,\cdots i_n}|i_1, i_2, \cdots i_n\rangle$$

$O(n)$ parameters

$O(2^n)$ parameters

A product state

A highly entangled state

A product state

$$|\Psi\rangle = |\phi_1\rangle|\phi_2\rangle\cdots\phi_n\rangle$$

$$|\phi_i\rangle = a_i|0\rangle + b_i|1\rangle$$

number of parameters $= 3n$

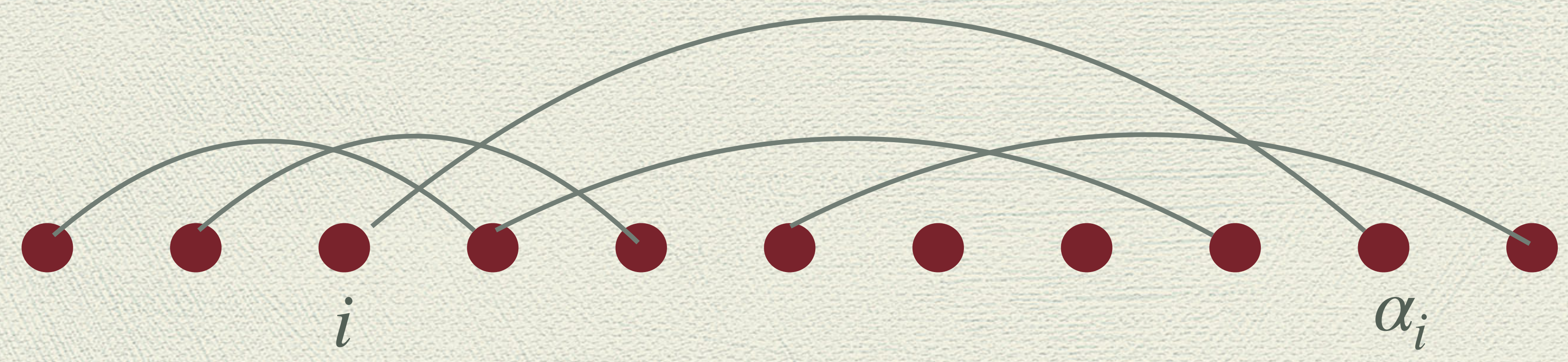
$$|\Psi\rangle = |\phi_{12}\rangle|\phi_{34}\rangle\cdots\phi_{n-1,n}\rangle$$

$$|\phi_{12}\rangle = a_{12}|00\rangle + b_{12}|01\rangle + c_{12}|10\rangle + d_{12}|11\rangle$$

number of parameters $= 6 \times n/2 = 3n$

A more complicated state

$\sum_{pairs}$



$$|\Psi\rangle = \sum_{pairs} \prod_{i,\alpha_i} |\phi_{i,\alpha_i}\rangle$$

$$\text{number of parameters} = \frac{n(n-1)}{2} \times 6 = O(n^2)$$

An even more complicated state

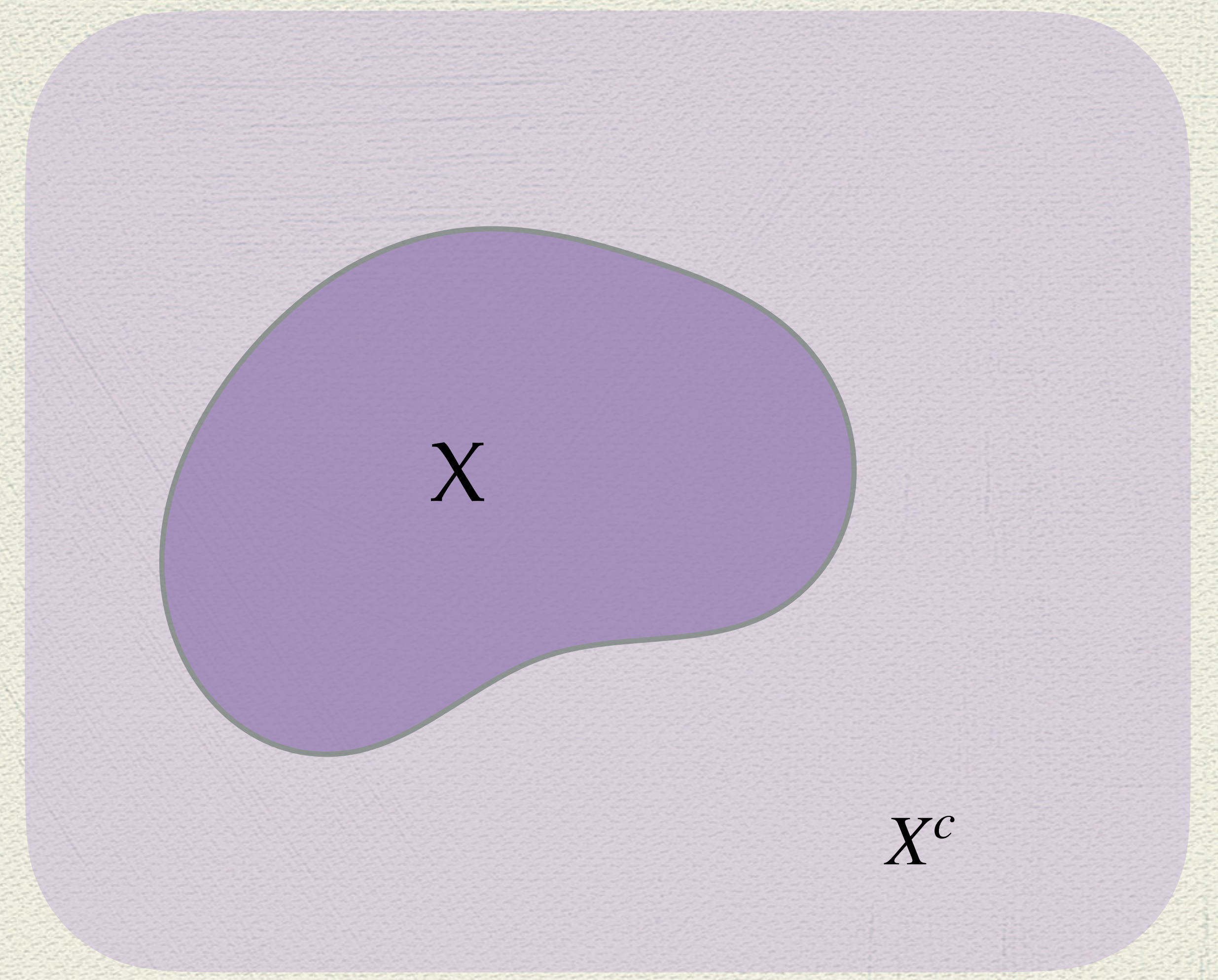
$$|\Psi\rangle = \sum_{pairs} \prod_{i,\alpha_i,\beta_i} |\phi_{i,\alpha_i,\beta_i}\rangle$$

$$\text{number of parameters} = \frac{n(n-1)(n-2)}{3!} \times 14 = O(n^3)$$

A many body system with local interactions

What is the pattern of interaction between X and X^c?

$$H = \sum_{i,j} h_{i,j}$$



$V(X) =$ Number of qubits in X

$$2^{V(X)} = dimH_X$$

There is only short-range (nearest neighbor) interactions in H.

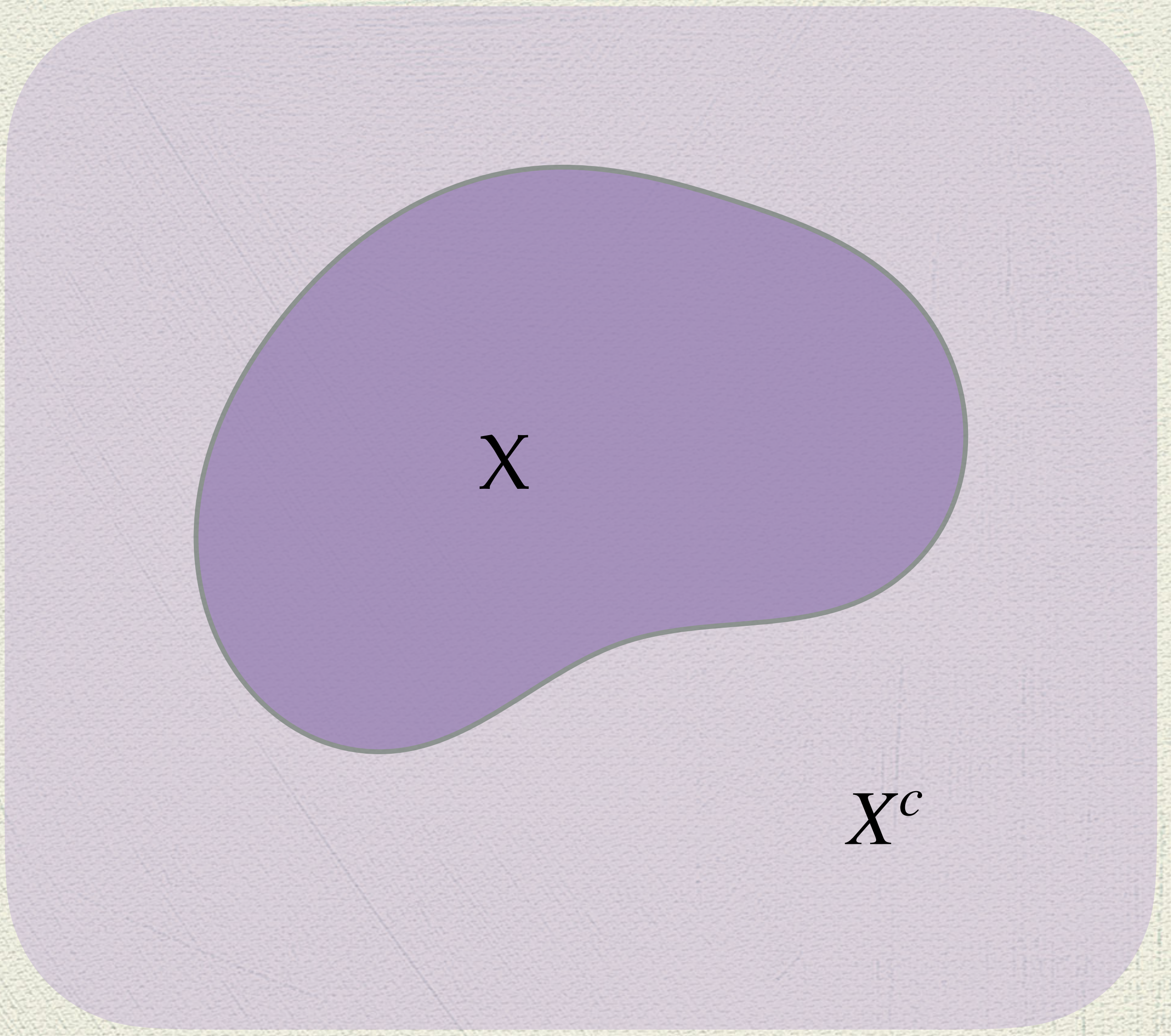
The Ground State

Schmidt Decomposition

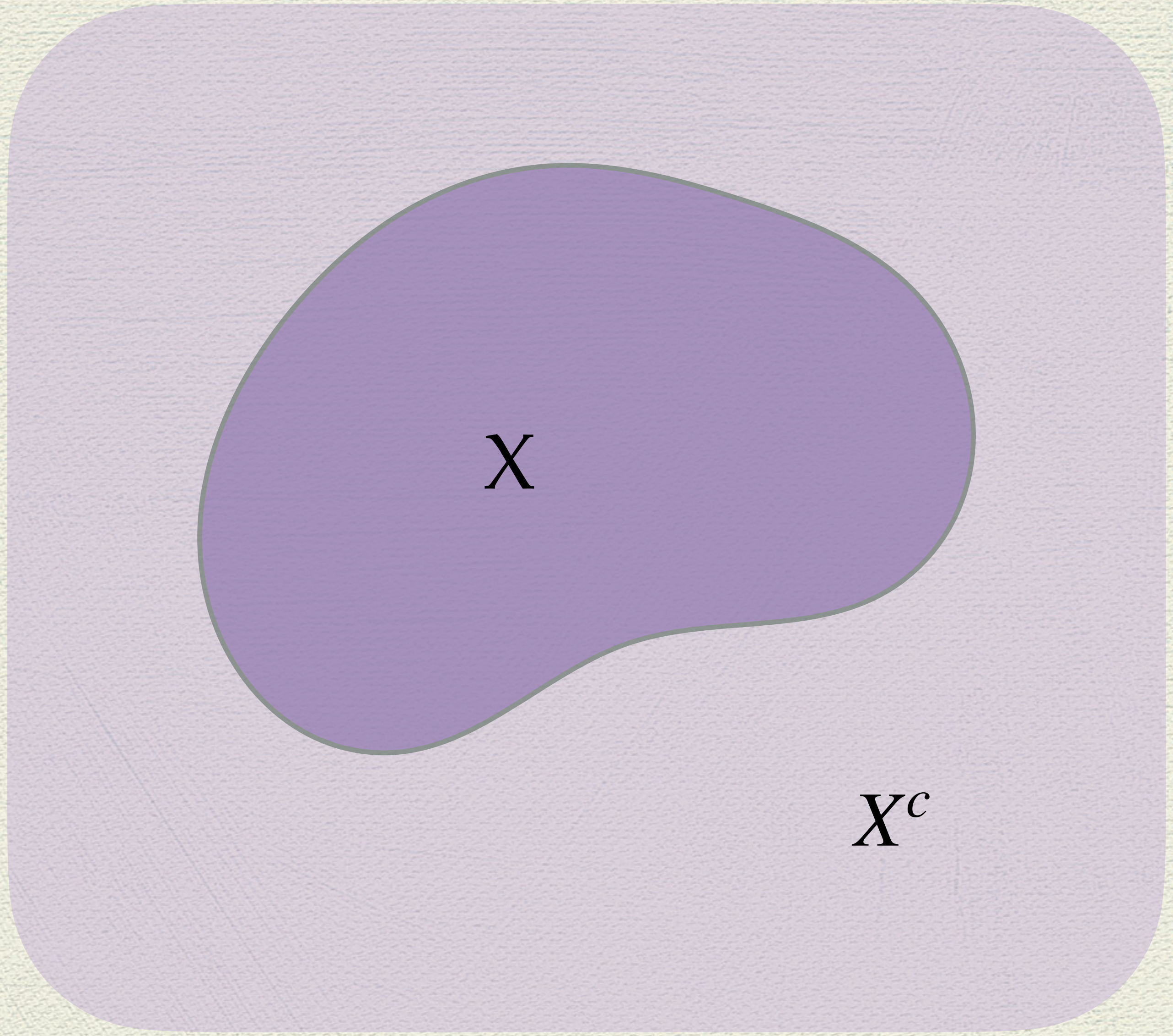
$$|\Psi\rangle = \sum_{\alpha=1}^{2^{V(X)}} \psi_\alpha |\alpha\rangle_X \otimes |\tilde{\alpha}\rangle_{X^c}$$

The Ground State can have long or short range entanglement or correlations.

Correlations are different from interactions

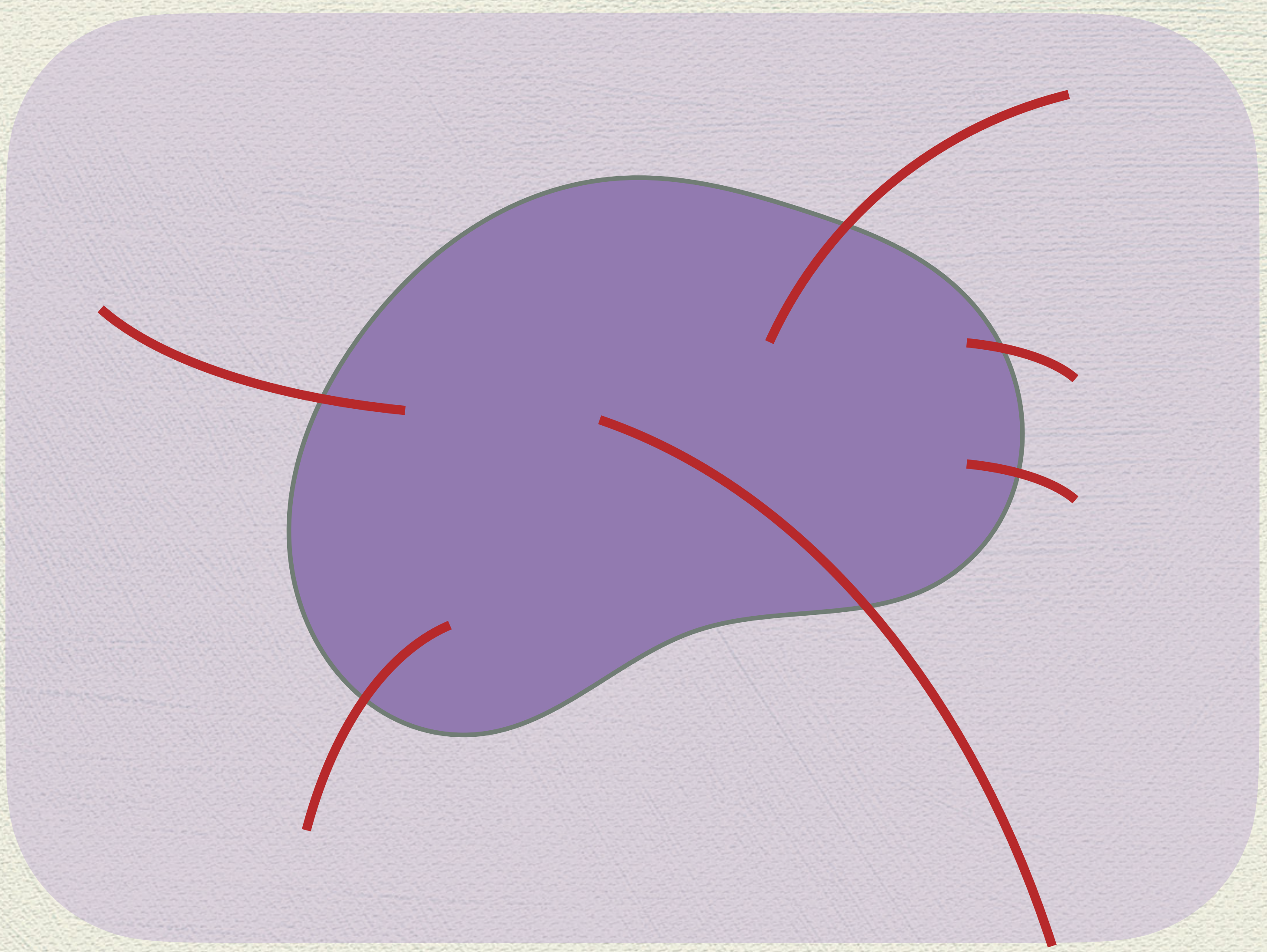


$$\rho_X = Tr_{X^c}|(\Psi\rangle\langle\Psi|)$$



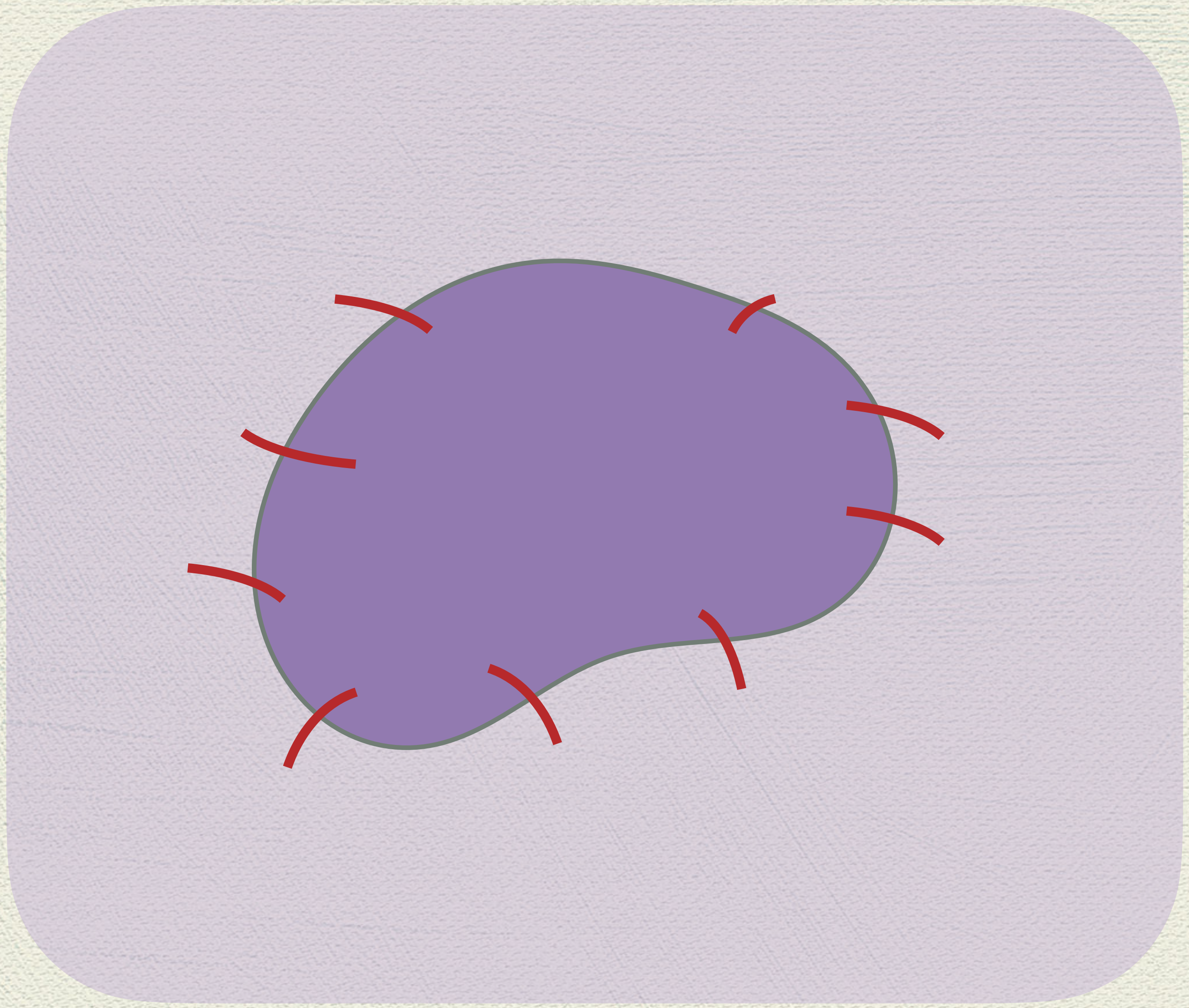
$$S(X) = -Tr(\rho_X \log \rho_X)$$

$$S(X) = -\sum_{\alpha=1}^{2^{V(X)}} |\psi_\alpha|^2 |\log|\psi_\alpha|^2 \longrightarrow 0 \le S(X) \le V(X)$$



If there is long range entanglement in the ground state → $S(X) \sim V(X)$

We say volume Law holds.



But for most systems,
there is only range entanglement → $S(X) \sim A(X)$

We say area law holds.

The ground state of most systems obey area law.

These types of groud states have a simple description.

The problem: Given a Hamiltonian H, find its ground state.

The Answer (Kitaev): It is an NP hard problem.

The importance of area law

These states have classical representations and their ground state can be found in polynomial time.

Question: When area law holds?

Answer: 1D Hamiltonians with energy gap.

Conjecture: This is true in any dimension

Area laws for the entanglement entropy – a review

J. Eisert,[1,2,3] M. Cramer,[3,4] and M.B. Plenio[3,4]

[1] Institute of Physics and Astronomy, University of Potsdam, 14469 Potsdam, Germany

[2] Institute for Advanced Study Berlin, 14193 Berlin, Germany

[3] Institute for Mathematical Sciences, Imperial College London, London SW7 2PG, UK

[4] Institut für Theoretische Physik, University of Ulm, 89069 Ulm, Germany

The ground states of systems with energy gap are best approximated by Matrix Product States (MPS).

$$\psi_{i_1,i_2,\cdots i_n} = Tr(A_{i_1}A_{i_2}\cdots A_{i_n})$$

For homogeneous systems

$$\psi_{i_1,i_2,\cdots i_n} = Tr(A_{i_1}^{(1)}A_{i_2}^{(2)}\cdots A_{i_n}^{(n)})$$

For inhomogeneous systems

For a spin 1/2 system: A_0 , A_1

For a spin 1 system: A_1 , A_0 , A_{-1}

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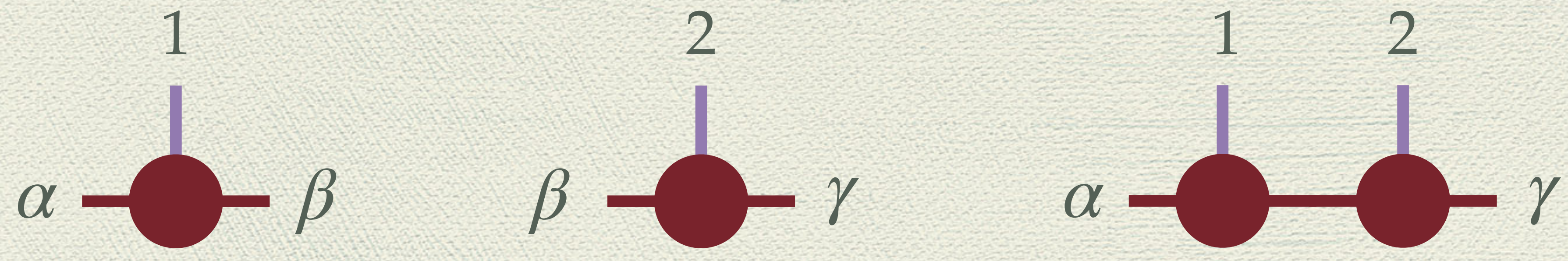
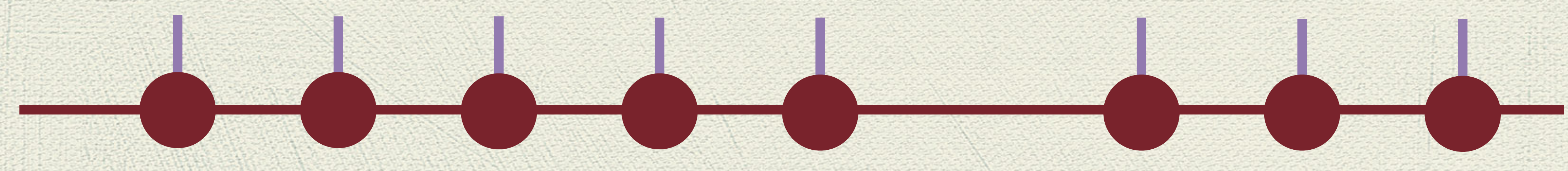
Graphical Representation



$A^{(1)}_{\alpha\beta}$ $\quad$ $A^{(2)}_{\beta\gamma}$ $\quad$ $(A^{(1)}A^{2})_{\alpha\gamma}$

A_{i_1} $\quad$ A_{i_2} $\quad$ A_{i_3} $\quad$ A_{i_n}



Number of parameters $= nD^2$

An example of an MPS

$$\langle\psi|\psi\rangle = Tr(E^N)$$

$$E = \sum_i A_i \otimes A_i^*$$

$$\langle\psi|\sigma_z|\psi\rangle = Tr(E_z E^{N-1})$$

$$E_z = A_0 \otimes A_0^* - A_1 \otimes A_1^*$$

$$\langle\psi|\sigma_x|\psi\rangle = Tr(E_x E^{N-1})$$

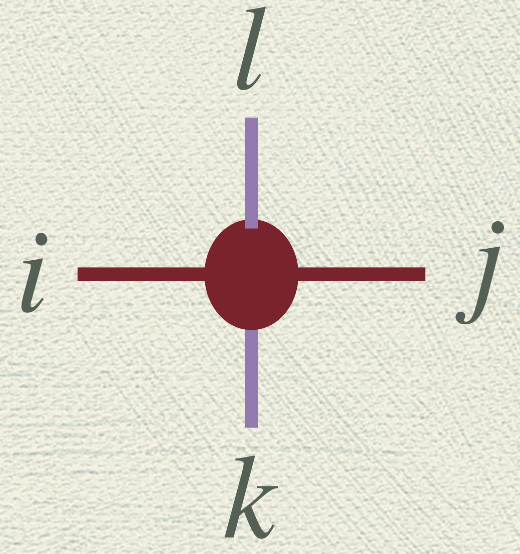
$$E_x = A_0 \otimes A_1^* + A_1 \otimes A_0^*$$

$$\langle\psi|\sigma_{z,1}\sigma_{z,k}|\psi\rangle = Tr(E_z E^{k-1} E_z E^{N-k-1})$$

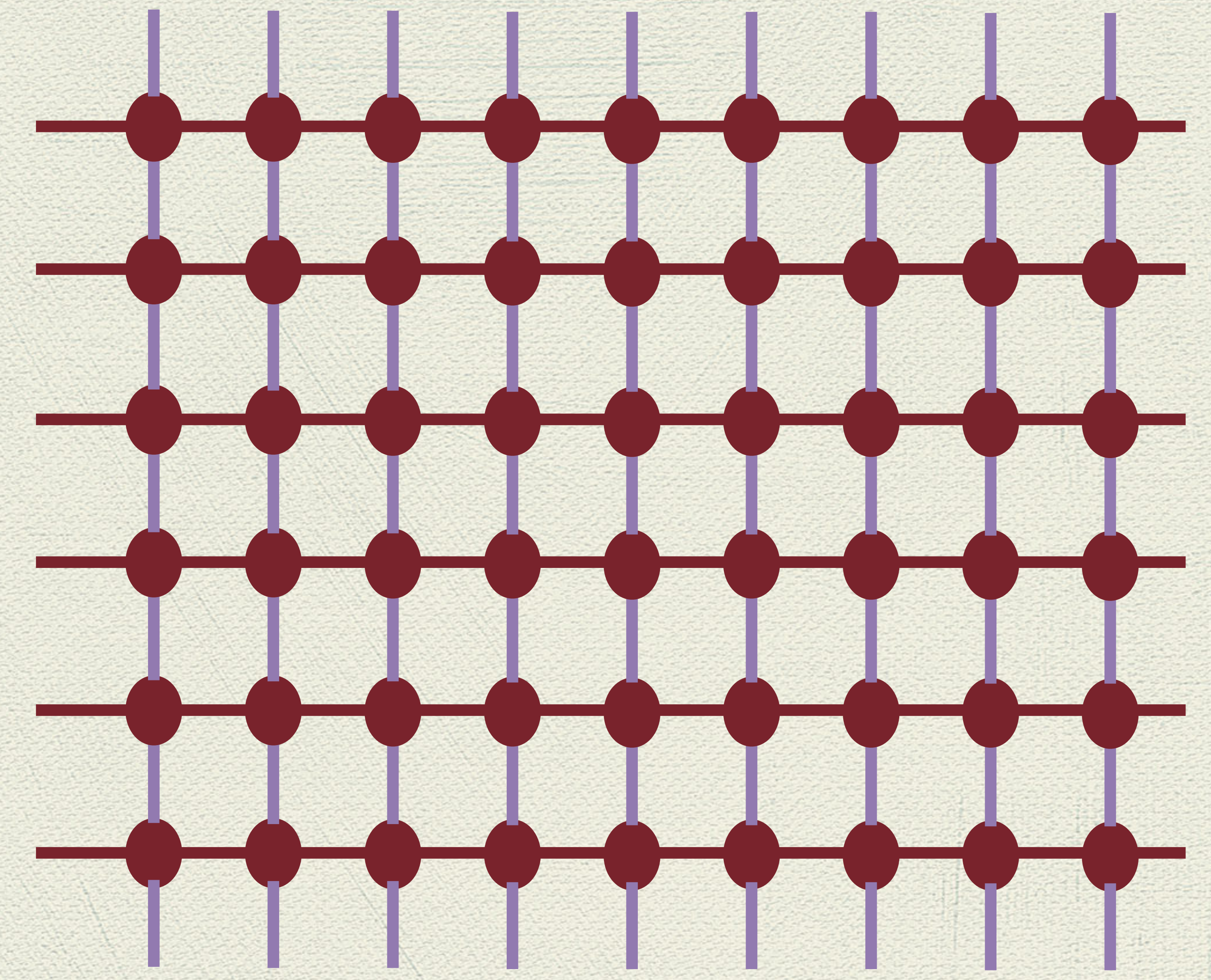
$$\langle\psi|\sigma_{z,j}\sigma_{z,k}|\psi\rangle \sim e^{-\frac{|j-k|}{\xi}}$$

$$\xi = \frac{1}{\log(\frac{\lambda_1}{\lambda_0})}$$

Tensor Networks (92-95)



$$T_{ij}^{kl}$$



Quantum Tensor Networks in a Nutshell

Jacob Biamonte[1,2,*] and Ville Bergholm[1,†]

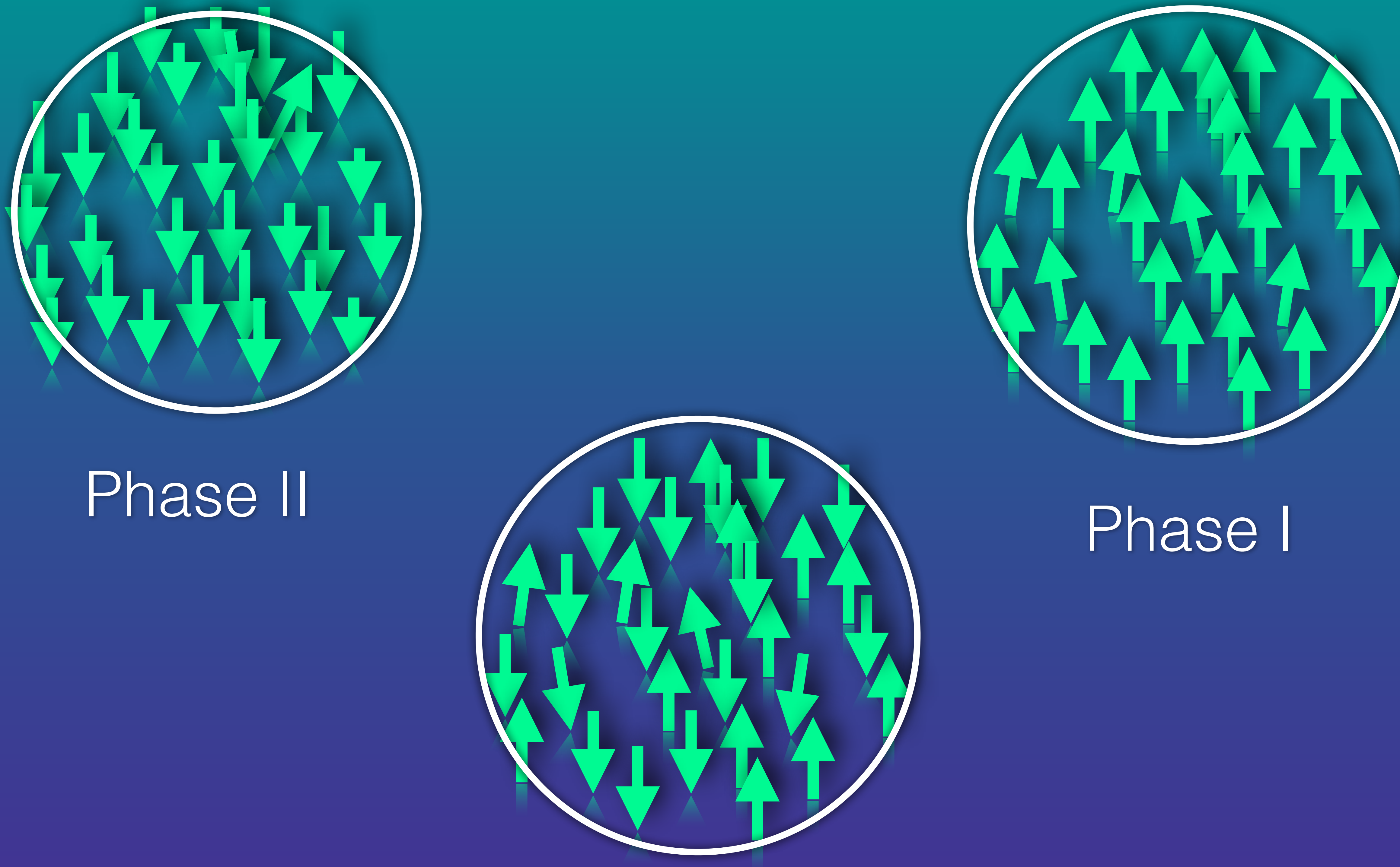
[1]*Quantum Software Initiative*
Skolkovo Institute of Science and Technology,
Skoltech Building 3, Moscow 143026, Russia
[2]*Institute for Quantum Computing*
University of Waterloo, Waterloo, N2L 3G1 Ontario, Canada

Topological phases of matter

Different phases of matter, like ice, water, and vapor are distinguished from each other by local observation.

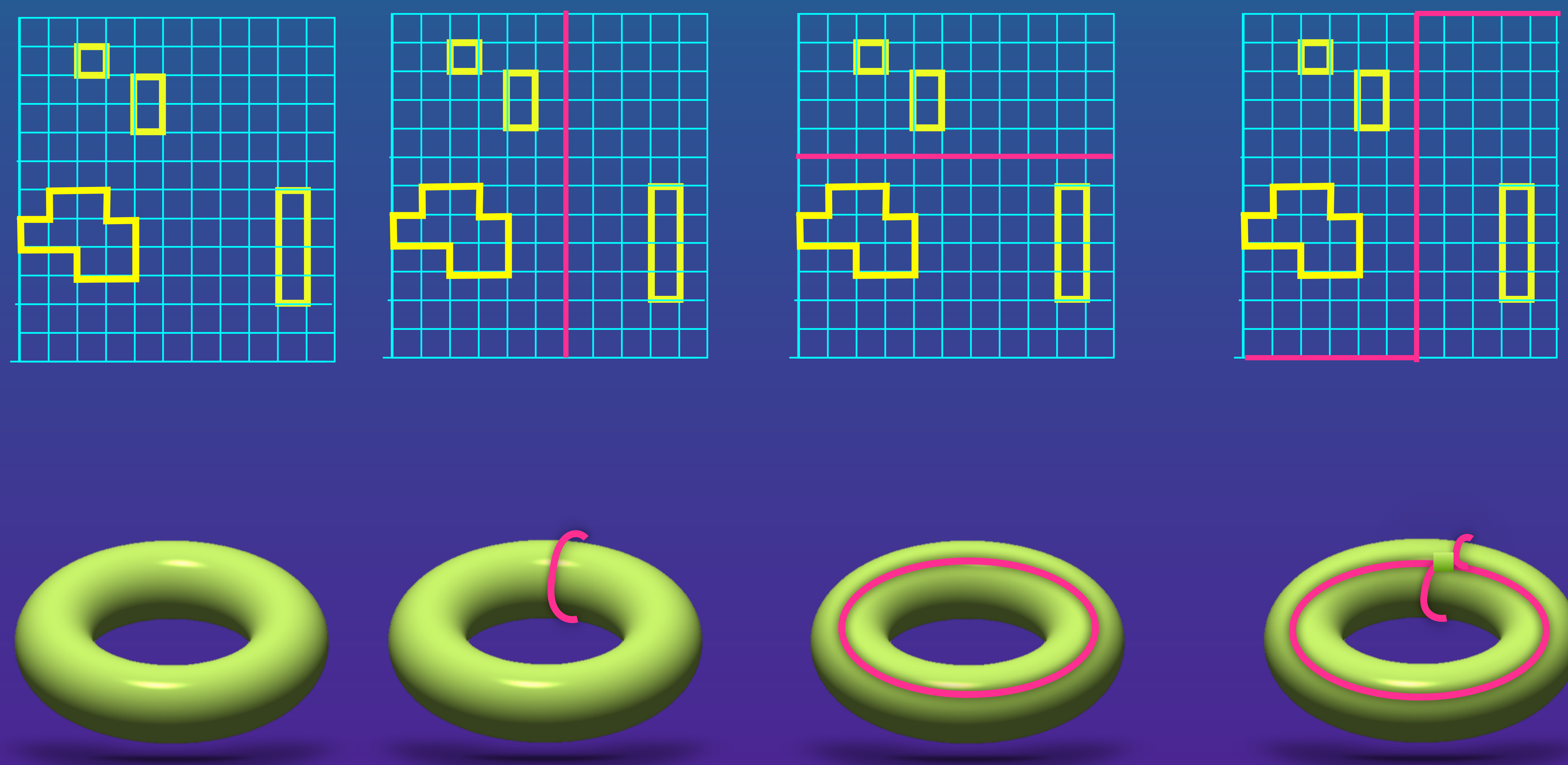
The same is true for phases like ferromagnetic phases and non-magnetic phases

Phase I



These three phases are distinguished by local observation.

Topological Phases cannot be distinguished by local observation



- Computational hardness of preparing ground states
- Entanglement dynamics in chaotic quantum systems
- Entanglement spreading